The theory of market maker

by

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Definitions

Market maker (MM) is a person, who always has at the Exchange Bid and Offer for certain commodity (1). Person from the definition means any member of the Exchange. Always means at the trading hours, and if traded new bid or offer should be put to the market in certain small time. Exchange means any trading platform.

Bid means will of the person to buy commodity.

Offer means will of the person to sell commodity.

Order is will of the person to buy or sell commodity. In every order is information for price, commodity, commodity we buy it for (usually currency), and volume.

Commodity means any type of commodity or financial instrument or currency. With commodity we shell understand also settlement and clearing options such as SPOT, Options or Futures.

Volume means how much/many commodity person is willing to buy or sell in an order.

Market maker

Preface

This document explains how MM makes money, formulas MM use to set order price and volume, and risks associated with it. Everybody already heard that MM makes a lot of money from the liquidity of the exchange with spread of bid and offer. This document explains how programmatically works MM and earns money. In whole text we will expect commodity to be EUA[1], and currency EUR. It is because there are no special risks with trading EUA such as delivery, quality and others.

Basics of earning money

If someone buy at lower price then he sell, then he earns money. It is not that easy, because if we are market maker, we cannot always sell our commodity at one price[2], we must move the price. If we move price too far, we could even lose money. For example, we buy 1000 EUA for 10 €, then we must sell it higher than 10 €.

F(x)

We will call F(x) formula to set the price of commodity at any time. At the beginning we shell know how much money (*cc*) we want to use, and how much commodity (*Q*) we want to use. Then from the formula we can get the price (*p*) and volume. When we will move cc and Q, we will receive new equilibrium price.

It is important for the formula to have

$$\lim_{x\to\infty} f(x) = 0$$

and

$$\lim_{x\to 0+} f(x) = \infty$$

.

One option to set f(x) is $p = f(cc, Q) = \frac{cc}{Q}$.

Other option to set f(x) is $p = f(cc, Q) = \frac{cc^n}{Q^n}$.

Other option is to create constant intervals, for example for

<0,q1> : $-\frac{MAXp-p1}{q1} * q + MAXp$

<q1,∞> : $-\frac{p1}{MAXq-q1} * q + p1 + \frac{p1}{MAXq-q1} * q1$

For simplicity we will use in further text formula $p = f(cc, Q) = \frac{cc}{Q}$

[1] European allowance to emit CO2

[2] Because our commodity volume is limited

When we start at certain *CCorig*, and *Qorig*, that will be constant, we can determine price for any quantity we currently have available. $p = f(q) = \frac{CCorig+g(q)}{Qorig+q}$ Now we need function g(q), that is recursion function of f(q) with 0 at the CCorig and Q orig.

MAXbid(q), MINoffer(q)

Now, when we have equilibrium price at f(q), we can set maximum bid and minimum offer functions.

$$MAXbid(q) = f(q) - BidZero(q) - \pi$$

$$MINoffer(q) = f(q) + OfferZero(q) + \pi$$

Functions ***BidZero(q)*** and ***OfferZero(q)*** are the most important functions in MM operations. If we set bid, and we sell commodity at certain price, we will have new equilibrium price f(q) that is lower. BidZero function represents zero profit for market maker to operate. It means, that if we are in the position of $f_0(q)$, then we bid and then sell back, we should get to previous position: $f_0(q)$. V should be traded volume.

$$f(q) = f(q) - BidZero(q) + OfferZero(q - V) = f(q)$$

$$f(q) = f(q) + OfferZero(q) - BidZero(q + V) += f(q)$$

Now we have 2 possibilities. We could have either constant trading volume, we could call it lot, or we could have constant spread. If we decide to have constant spread, then we must know how much volume we can trade.

BidZero(q), OfferZero(q)

Lets divide this section into 2 sections: Constant volume, and constant spread

Constant volume

We need to find a minimum spread (*sp*) we can trade for.

	Step 0 - basic	Step 1 – we buy/bid	Step 2 – we sell/offer
Money	Cc	$CC1 = CC - (\frac{CC}{Q} - sp) * q$	$CC2 = CC1 + (\frac{CC1}{Q1} + sp) * q$
Quantity	Q	Q1 = Q + q	Q2 = Q1 – q
Equilibrium price	F(x) = cc/Q	F(x) = CC1/Q1	F(x) = CC2/Q2
Volume	q	q	q
Bid	(CC/Q – sp)*q	(CC1/Q1 – sp) *q	(CC2/Q2-sp)*q
Offer	(CC/Q + sp)*q	(CC1/Q1 + sp)*q	(CC2/Q2+sp)*q

When we want CC2 to be equal to CC, then $CC = CC1 - (\frac{CC1}{Q1} - sp * q)$, then

$$CC = CC - \left(\frac{CC}{Q} - sp\right) * q + (\frac{CC - (\frac{CC}{Q} - sp) * q}{Q1} + sp) * q$$

$$0 = -\left(\frac{CC}{Q} * q - sp * q\right) + (\frac{CC - (\frac{CC}{Q} * q - sp * q)}{Q1} + sp) * q$$

$$0 = -\frac{\mathrm{CC}}{\mathrm{Q}} * \mathrm{q} + \mathrm{sp} * \mathrm{q} + \left(\frac{\mathrm{CC} - \frac{\mathrm{CC}}{\mathrm{Q}} * \mathrm{q} + \mathrm{sp} * \mathrm{q}}{\mathrm{Q} + \mathrm{q}} + \mathrm{sp} \right) * \mathrm{q}$$

$$q > 0; /q$$

$$0 = -\frac{\mathrm{CC}}{\mathrm{Q}} + \mathrm{sp} + \frac{\mathrm{CC} - \frac{\mathrm{CC}}{\mathrm{Q}} * \mathrm{q} + \mathrm{sp} * \mathrm{q}}{\mathrm{Q} + \mathrm{q}} + \mathrm{sp}$$

$$0 = \frac{-\frac{\mathrm{CC}}{\mathrm{Q}} * (\mathrm{Q} + \mathrm{q}) + \mathrm{sp} * (\mathrm{Q} + \mathrm{q}) + \mathrm{CC} - \frac{\mathrm{CC}}{\mathrm{Q}} * \mathrm{q} + \mathrm{sp} * \mathrm{q} + \mathrm{sp} * (\mathrm{Q} + \mathrm{q})}{\mathrm{Q} + \mathrm{q}}$$

$$0 = \frac{-\frac{\mathrm{CC}}{\mathrm{Q}} * \mathrm{Q} - \frac{\mathrm{CC}}{\mathrm{Q}} * \mathrm{q} + \mathrm{sp} * \mathrm{Q} + \mathrm{sp} * \mathrm{q} + \mathrm{CC} - \frac{\mathrm{CC}}{\mathrm{Q}} * \mathrm{q} + \mathrm{sp} * \mathrm{q} + \mathrm{sp} * \mathrm{Q} + \mathrm{sp} * \mathrm{q}}{\mathrm{Q} + \mathrm{q}}$$

$$0 = \frac{-\mathrm{CC} - 2 * \frac{\mathrm{CC}}{\mathrm{Q}} * \mathrm{q} + 2 * \mathrm{sp} * \mathrm{Q} + \mathrm{CC} + 3 * \mathrm{sp} * \mathrm{q}}{\mathrm{Q} + \mathrm{q}}$$

$$Q + q > 0; /(Q + q)$$

$$0 = -2 * \frac{\mathrm{CC}}{\mathrm{Q}} * \mathrm{q} + 2 * \mathrm{sp} * \mathrm{Q} + 3 * \mathrm{sp} * \mathrm{q}$$

$$\mathrm{sp} * (2 * \mathrm{Q} + 3 * \mathrm{q}) = 2 * \frac{\mathrm{CC}}{\mathrm{Q}} * \mathrm{q}$$

$$\mathrm{sp} = \frac{2 * \mathrm{CC} * \mathrm{q}}{\mathrm{Q} * (2 * \mathrm{Q} + 3 * \mathrm{q})}$$

When we sell at the step 1, then

	Step 0 - basic	Step 1 – we sell/offer	Step 2 – we buy/bid
Money	Cc	$\mathrm{CC1} = \mathrm{CC} + (\frac{\mathrm{CC}}{\mathrm{Q}} + \mathrm{sp}) * \mathrm{q}$	$CC2 = CC1 - (\frac{\mathrm{CC1}}{\mathrm{Q1}} - \mathrm{sp}) * \mathrm{q}$
Quantity	Q	Q1 = Q - q	Q2 = Q1+ q
Equilibrium price	F(x) = cc/Q	F(x) = CC1/Q1	F(x) = CC2/Q2
Volume	q	q	q
Bid	(CC/Q – sp)*q	(CC1/Q1 – sp) *q	(CC2/Q2-sp)*q
Offer	(CC/Q + sp)*q	(CC1/Q1 + sp)*q	(CC2/Q2+sp)*q

Then to get zero profit, we need to place cc2=cc: $CC = CC1 - (\frac{\mathrm{CC1}}{\mathrm{Q1}} - \mathrm{sp}) * \mathrm{q}$

Then

$$\mathrm{sp} = \frac{2 * \mathrm{CC} * \ \mathrm{q}}{\mathrm{Q} * (2 * \mathrm{Q} - 3 * \mathrm{q})}$$

This means, that if we have 1 000 000 tn of EUA, and 10 000 000 EUR of money, and would like to trade at lot 1000 tn, we could be market maker minimum spread :

$$\mathrm{Spread} = \mathrm{sp} = \frac{2 * 10\,000\,000 * \ 1\,000}{1\,000\,000 * (2 * 1\,000\,000 - 3 * 1\,000)} = \frac{20000000000}{1997000000000} = \ 0{,}01 \text{ EUR}$$

Constant spread

We need to find out what volume we can trade at specific CC/Q and spread.

	Step 0 - basic	Step 1 – we sell/offer	Step 2 – we buy/bid
Money	Cc	$\mathrm{CC1} \ = \mathrm{CC} + (\frac{\mathrm{CC}}{\mathrm{Q}} + \mathrm{sp}) * \mathrm{q}$	$CC2 = CC1 - (\frac{\mathrm{CC1}}{\mathrm{Q1}} - \mathrm{sp}) * \mathrm{q}$
Quantity	Q	Q1 = Q - q	Q2 = Q1+ q
Equilibrium price	F(x) = cc/Q	F(x) = CC1/Q1	F(x) = CC2/Q2
Volume	q	q	q
Bid	(CC/Q – sp)*q	(CC1/Q1 – sp) *q	(CC2/Q2-sp)*q
Offer	(CC/Q + sp)*q	(CC1/Q1 + sp)*q	(CC2/Q2+sp)*q

We need to set CC2 = CC:

$$CC = \mathrm{CC} - \left(\frac{\mathrm{CC}}{\mathrm{Q}} - \mathrm{sp}\right) * \mathrm{q} + (\frac{\mathrm{CC} - (\frac{\mathrm{CC}}{\mathrm{Q}} - \mathrm{sp}) * \mathrm{q}}{\mathrm{Q1}} + \mathrm{sp}) * \mathrm{q}$$

$$0 = \ -2 * \frac{\mathrm{CC}}{\mathrm{Q}} * \ \mathrm{q} + 2 * \mathrm{sp} * \mathrm{Q} \ + 3 * \mathrm{sp} * \mathrm{q}$$

$$2 * \frac{\mathrm{CC}}{\mathrm{Q}} * \ \mathrm{q} - 3 * \mathrm{sp} * \mathrm{q} = \ 2 * \mathrm{sp} * \mathrm{Q}$$

$$\mathrm{q} * (2 * \frac{\mathrm{CC}}{\mathrm{Q}} - 3 * \mathrm{sp}) = \ 2 * \mathrm{sp} * \mathrm{Q}$$

$$\mathrm{q} = \frac{2 * \mathrm{sp} * \mathrm{Q}}{2 * \frac{\mathrm{CC}}{\mathrm{Q}} - 3 * \mathrm{sp}}$$

If we first buy/bid, then

$$\mathrm{q} = \frac{2 * \mathrm{sp} * \mathrm{Q}}{2 * \frac{\mathrm{CC}}{\mathrm{Q}} + 3 * \mathrm{sp}}$$

This result means that if we have 10 000 000 eur, and 1 000 000 EUA, and we want constant spread 0.01 eur, then we should make offer at 9.99 EUR at volume 998 and bid at price 10.01with volume 1001 EUA.

Market maker order

For the market maker it is the most easiest if the trading platform has integrated market maker orders. TradeSys trading platform at Commodity Exchange Bratislava has this order fully implemented and it's working.

Risks associated with market maker

Market maker makes money with the liquidity. If the market goes up and down, everything for the market maker works fine. The primary risk for market maker is when the commodity looses value, or the currency looses value. Then the market would move to zero price, and volume would be great. If the currency collapse, then the volume would go to zero, and money would be great.

Other risk could be considered trading platform. If the liquidity is very low, then market maker does not make any money.

Bibliography

1. **CEB.** *Exchange rules.* Bratislava : Commodity Exchange Bratislava, JSC, 2009.